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The Number Irreducible Constituents Permutation Representation of G

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ABSTRACT: et $D_1,\ldots,D_r$ the different irreducible constituents of the permutation representation G^* of G. In addition let f_i be the degree of $D_i(i=1,\ldots,r)$ and let $\chi_i = Tr(D_i)$ be the character of D_i. Let the numbering be chosen so that D_1 is the identity representation. Then number the irreducible constituents D_i of G^* such that $f_2 = p$ and the representations $D_3,\ldots,D_r$ are conjugate. In particular $f_3 = \ldots = f_r = f$, and f divides p-1.

***Keywords*:** permutation representation, orbits, irreducible constituents.

INTRODUCTION

In 1943. R. Brauer studied about permutation groups and finid the permutation groups of prime degree and related classes of Groups (See (4)), From years 1906 to 1936. W.A.Manning studied about primitive groups and finding the primitive groups of classes six, ten , twelve and fifteen . And in 1906, W.Burnside introduced d researched the a bout transitive groups, of prime degree (See (2)), and in 1921, he worked about the certain simply- transitive permutation group and obtained a beautiful consequences (See (3)). (See(11),(12),(13),(14),(15),(16),(17) and (18)). In 1937 J.S. Frame determined the degrees of the irreducible components of simply transitive permutation groups, and in 1941, he obtained the double cosets of a finite groups (See(5) and (6)). And also in 1952, he finding the irreducible representation extracted from two permutation groups (See (7)). G.A.Miller(1897&1915),(See (19) &(20)), E.T.Parker (1954),(See (21)), M. Suzuki(1962), (See(23)), J.G.Thompson (1959), (See (24)), M.J.Weiss(1928), (See (25)&(26)), H.Wielandt (1935 & 1956) (See (27)&(28)) and H.Zassenhaus(1935), (See (30)) are studied about transitive and primitive groups and their obtaind the beautiful and more consequence. Now in this paper we will prove number the irreducible constituents D_i of G^* such that $f_2 = p$ and the representations $D_3,\ldots,D_r$ are conjugate. In particular $f_3 = \ldots = f_r = f$, and f divides p-1.

2. Periliminares

In this chapter we study the notations, elementary properties ,lemmas and theorems, whose we will used in chapter3.

2.1. Elementary notions and definitions

Let Ω be a finite set of arbitrary elements which for natural numbers 1,2,…,n as the points and Δ subset of Ω . Then a permutation on Ω is a one-to-one mapping of Ω onto itself. We denote the image of the point $\alpha \in \Omega$ under the permutation p by α^p.We write $P = \begin{pmatrix} 1 & 2 & \ldots & n \\ 1^p & 2^p & \ldots & n^p \end{pmatrix} = \begin{pmatrix} \alpha \\ \alpha^p \end{pmatrix}$. We define the product pq of two permutations p and q on Ω by the formula $\alpha^{pq} = (\alpha^p)^q$. trivialy pq is again a permutation on Ω.

With respect to the operation above, all the permutations on Ω form a group, the symmetric group S^G. Let G be a permutation group on Ω, in short $G \le S^{\Omega}$. We say that a set $\Delta \subseteq \Omega$ is a fixed block of G or is fixed by G if $\Delta = \Delta^G$. Then each $g \in G$ induces a permutation on Δ which we denote by g^{Δ}. We call the totality of g^{Δ}'s formed for all $g \in G$ the constituent G^{Δ} of G on Δ (for example $G = G^{\Delta}$). G^{Δ} is a permutation group on Δ. Obviously the mapping $g \to g^{\Delta}$ is a homomorphism : $G \xrightarrow{\sim} G^{\Delta}$. If this mapping is an isomorphism, that is, $|G^{\Delta}| = |G|$, then the constituents G^{Δ} is called faithful. Every group G on Ω has the trivial fixed blocks ϕ on Ω. If it has no others it is called transitive. Otherwise it is called intransitive. Accordingly, a constituent G^{Δ} is transitive precisely when Δ is a minimal fixed block ($\Delta \ne \phi$). In this case Δ is called an orbit or set of transitivity of G. Every permutation g on Ω can beregarded in the following way as a liner substitution in $|\Omega|$=n variables . the variables $X_1, \dots, X_n$ are taken as points. We form colum vectors $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \; x^g = \begin{bmatrix} x_1^g \\ \vdots \\ x_n^g \end{bmatrix} = g^*_x$ where $g^* = \left(\delta_{\alpha^g, \beta}\right)_{\alpha, \beta = 1, \dots, n}$ is the n by n matrix corresponding to the linear transformation $x \to x^g$. ($\delta_{\alpha\beta} = \begin{cases} 1 & , \; \alpha = \beta \\ \circ & , \; \alpha \ne \beta \end{cases}$ is the well known Kronecker symbol). We call g* the permutation matrix corresponding to g. such a matrix contains exactly one 1 in each row and column and zeros every where else. In addition, every permutation matrix g* is orthogonal , i.e., the transpose $g^{*\prime}$ of g* is identical with its inverse: $g^{*\prime} = g^{*-1}$, we obtain a faithful representationof S^G by $g \to g^*$. Now let $G \le S^G$. By G* we denote the group of all matrices g* with $g \in G$. Obviously G* is isomorphic to G. We call G* the permutation representation of G.

2.2.Theorem

(See (22)). If a transitive permutation group G is regarded as a matrix group G*, then the matrices which commute with all the matrices of G* form a ring V=V(G). We call V "the centralizer ring corresponding to G". V is a vector space over the complex number field which has the matrices $B(\Delta)$ corresponding to the orbits Δ of G_1 as a linear basis. In particular, the dimension of V coincides with the number k of orbits of G_1.

Proof

See(29) , Theorem 28.4)

Let $D_1, \dots, D_r$ be the different irreducible representations appearing in G* where D_1 is the identity representation. In the following we always denote by f_i the degree of D_i (i=1,…,r), and by e_i the multiplicity of D_i in G*. In particular, we have $e_1 = f_1 = 1$ and $\sum_i e_i f_i = n$, the reduction of G* gives for an appropriately chosen unitary n by n matrix U :

$$U^{-1} G^* U = [D_1, \underbrace{D_2, \dots, D_2}_{e_2}, \dots, \underbrace{D_n, \dots, D_n}_{e_r}].$$

2.3.Proposition

Let M be the n by n matrix whose elements are all 1.then $\sum_{\Delta} B(\Delta) = M.$ (Here the summation is over all orbits of G_i)

Proof

See ((29), proposition 28.2) .

2.4.Theorem

V is commutative if and only if all the e_i=1.

Proof

See ((29) , Theorem 29.3)

2.5.Theorem (29.8)

V is commutative if and only if the class matrices $E_i = \sum_{g \in C_i} g^*$ (i = 1,..,n) whose C_i be the ith class of conjugate elements of G, generate V, i.e., when each $B \in V$ has a (not necessarily unique) representation $B = \sum_{i=1}^{r} z_i E_i$.

Proof

See ((29), Theorem 29.8)

2.6.Theorem

Let Γ and Δ be two orbits of G_1. then $Tr(B(\Gamma)'B(\Delta)) = \begin{cases} \circ & , \quad \Gamma \neq \Delta \\ |\Gamma| n & , \quad \Gamma = \Delta \end{cases}$

Proof

See ((29), Theprem 28.10)

2.7.Difinition

By a Burnside-group (in short : B-group) we mean an abstract finite group H with the property that every primitive group containing the regular representation of H as a transitive subgroup is doubly transitive. (See (28), p.343).

2.8.Theorem

(See (27)).Every cyclic group of composite order is a B-group.

Proof

See ((29), Theorem 25.3)

2.9. Theorem

(See (10)): G is doubly transitive. If in addition G_Δ is primitive on Γ, then G is even doubly primitive (Jordan theorem).

Proof

See ((29), Theorem 13.1)

2.10. Difinition

A permutation group G on Ω is called semiregular if, for each $\alpha \in \Omega, G_\alpha = 1;$ and G is called regular if it is semiregular and transitive, Accordingly, every regular group is also semiregular and subgroups as well as constituents of semiregular groups are semiregular 1 is semiregular. In the case of semiregular groups, the degree and minimal degree coincide.

2.11. Proposition

The order of a semiregular group is a divisor of its degree. A transitive group is regular if and only if its order and degree are equal.

Proof

See ((29), Proposition 4.2)

2.12. Theorem

Every normal subgroup $\neq 1$ of a primitive group is transitive.

Proof

See ((29), Theorem8.8)

2.13.Theorem

The representation module associated with G* contains a one-dimensional invariant subspace corresponding to the identity representation, namely, the one generated by $\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$, and because of the transitivity of G it contains no others. The identical representation therefore appears in G* with multiplicity exactly 1.

Proof

See ((29)), theorem 29.1)

2.14. Theorem

G is doubly transitive if and only if dim V(G)=2. In this case G* has exactly two irreducible constituents. In particular, we have r=k and V(G) commutative.

Proof

See((29), Theorem 29.9)

2.15. Theorem

(See (1), p.341). Every nonsolvable transitive group of prime degree is doubly transitive.

Proof

See ((29), Theorem 11.7 and (1), p. 341)

2.16. Proposition

Every abelian group G transitive on Ω is regular. G is its own centralizer in S^{Ω}.

Proof

See ((29), Proposition 4.4)

2.17. Proposition

If G is primitive on Ω and α and β are different points of Ω, Then either $G_{\alpha} \neq G_{\beta}$ or G is a regular group of prime degree.

Proof

See ((29), Proposition 8.6)

2.18. Theorem

Let G betransitive on Ω, |G| not a prime number, $\alpha \in \Omega,\ \beta \in \Omega, \alpha \neq \beta$. Let G have a subgroup H intransitive on Ω with the properties $\beta_{\alpha}^{G} \subseteq \beta^{H}$ and $\left|\beta^{H}\right| \leq \left|\alpha^{H}\right|$. Then G is imprimitive and $\left|\beta^{H}\right| = \left|\alpha^{H}\right|$.

Proof

See ((29), Theorem 27.5).

2.19. Theorem

Paired orbits have the same length.

Proof

See ((29), Theorem 16.3).

2.20. Difinition

Let G is transitive and consider the orbits of G_1. With each of these orbits Δ (including the trivial $\Delta=\{1\}$) we associate in the following way a matrix $B(\Delta)=\left(V^{\Delta}_{\alpha,\beta}\right)$, $\alpha,\beta=1,...,n$, with elements:
$V^{\Delta}_{\alpha,\beta}=\begin{cases}1, & if\ there exists\ g\in G\ and\ \delta\in\Delta\ with\ 1^{g}=\beta\ and\ \delta^{g}=\alpha.\\ 0, & otherwise\end{cases}$

Thus, in the first colum of $B(\Delta)$ we have exactly those $V^{\Delta}_{\alpha,\beta}=1$ for which $\alpha\in\Delta$ holds. If $\Gamma\neq\Delta$, the ones of $B(\Gamma)$ and $B(\Delta)$ do not occur in the same place. On the other hand , for each place (α,β) there is an orbit Δ of G, (namely, the one in which the $\alpha^{g^{-1}}$ with $1^{g}=\beta$ lies) such that $B(\Delta)$ has 1 in this position.

2.21. Theorem

The matrices corresponding by definition 2.18 to paired orbits of G_1 are transposes: $B(\Delta')=(B(\Delta))'.$

Proof

See ((29), Theorem 28.9)

2.22.Theorem

If G_α has an orbit Γ with $|\Gamma|=2$, then G contains a regular normal subgroup R of index 2. G is a Frobenius group.

Proof

See ((29), Theorem 18.7)

2.23. Theorem

(A) If the irreducible constituents of G* are all different, i.e., if all the multiplicities e_i=1, then the rational number

$$q=n^{k-2}\prod_{i=1}^{k}\frac{n_i}{f_i}$$

is an integer.

(B) If in addition the k numbers n_i are all different, then q is a square.

(C) If the irreducible constituents of G* all have rational characters, then q is a square. The hypothesis is always fulfilled if the degrees f_i are all different.

Proof of 2.23.(A)

It suffices to show that q is an algebraic integer. The notation of the preceding section is continued.
Let U again be the unitary transformation matrix introduced in §2. From the hypothesis e_i=1 it follows that every matrix $M=U^{-1}BU$ with $B\in V$ has diagonal form. Because $B_i=B(\Delta_i)\in V(G)$ we have in particular
$M_i=U^{-1}BU=[w_{1i},w_{2i}\Im_{f_2},...,w_{ki}\Im_{f_k}]$, which $\Im_{f_i}$ is the f_i by f_i identity matrix.

Let w_i be the diagonal elements of the matrix $M=U^{-1}BU$, for arbitrary $B\in V$. We put

$$B=\sum_i z_iB_i,\qquad z=\begin{pmatrix}z_1\\ \vdots\\ z_k\end{pmatrix},\quad w=\begin{pmatrix}w_1\\ \vdots\\ w_k\end{pmatrix},$$

$N=[1,n_2,...,n_k]$, $F=[1,f_2,...,f_k]$, and I=(w_{ij}); i,j=1,…,k. From $M=U^{-1}BU$ it follows that $\bar{M}'M=U^{-1}\bar{B}'BU$, since U was assumed unitary. With the aid of 2.4 we now obtain

$$\bar{z}'Nnz=\sum_{i=1}^{k}\bar{z}_i z_i n_i n=\sum_{i,j}\bar{z}_i z_j Tr(\bar{B}_i' B_j)=Tr(\bar{B}'B)=Tr(\bar{M}'M)=\sum_i \bar{w}_i w_i f_i=\bar{w}'Fw.$$

Because $w_i=\sum_j z_j w_{ij}$, i.e., $w=Iz$, we therefore have $Nn=\bar{I}'FI$. By taking the determinant

we get $n^k\prod_i n_i=|Nn|=|F|\,|\bar{I}'I|=\prod_i f_i\,|\bar{I}'|\,|I|.$

The w_{ij}, as eigenvalues of the matrix B_j which has integer coefficients, are algebraic integers, and therefore **| I |** and $|\bar{I}|$ are also algebraic integers.

We wish to show that $|I|$ is divisible by n. by 2.3, $\Sigma_j B_j$=M where M is the n by n matrix consisting of n^2 ones. M has the eigenvalue n occurring with multiplicity 1 belonging to the eigenvector $\begin{pmatrix}1\\ \vdots\\ 1\end{pmatrix}$. Its remaining eigenvalues are 0. In the diagonal matrix $\Sigma_j M_j$, n therefore appears exactly once, the remaining elements being 0. Therefore $\Sigma_j w_{ij}=n$ for i=1 and =0 for the remaining. This implies

$$|I|=\begin{vmatrix} n & w_{12} & \cdots & w_{1k}\\ 0 & w_{22} & \cdots & w_{2k}\\ \vdots & & & \\ 0 & w_{k2} & \cdots & w_{kk}\end{vmatrix}\equiv 0 \quad (mod\, n).$$

Hence $q=n^{-2}|\bar{I}|\,|I|$ is an algebraic integer, hence also a rational integer.

Proof of 2.23(C). (a)

Because of the hypothesis $e_1=\cdots=e_k=1$, the commutativity of V follows by 2.4. Theorem 2.5 yields the existence of k class matrices $C_1,...,C_k$ and of complex numbers x_{ij} such that $B_i=\sum_{j=1}^{k}x_{ij}C_j \quad (i=1,...,k).$

Conversely by 2.2 there are also x'_{ij} with $C_i=\sum_{j=1}^{k}x'_{ij}B_j$. The x_{ij} are, by well-known theorems of linear algebra, uniquely determined and rational, since the matrices B_i and C_j are rational.

(b) By hypothesis all the irreducible characters appearing in G* are rational. Thus the matrics $U^{-1}C_ju$ appearing in the proof of 2.5 are also rational. By (a) the matrices $M_i=U^{-1}B_iU=\sum_i x_{ij}U^{-1}C_jU$ are then rational. The w_{ij} are therefore rational. Since the w_{ij} were already in the proof of 2.23 (A) shown to be algebraic integers, they are rational integers. $|J|=|w_{ij}|$ is therefore a rational integer. Since n divides $|J|$, $n^{-1}|J|$ is also a rational integer, and q is therefore a square as was asserted.

(c) The hypothesis that all the irreducible characters appearing in G^* are rational is fulfilled if the degrees f_i of the irreducible constituents of G^* are all different. For since G* is rational, with each irreducible representation D_i all representations conjugate to it appear in G^*. Because all of the f_i are different, these coincide with D_i, and X_i is therefore rational.

3. Main Result

In this chapter we prepare the proof number the irreducible constituents D_i of G^* such that $f_2 = p$ and the representations $D_3, ..., D_r$ are conjugate. In particular $f_3 = ... = f_r = f,$ and f divides p-1.

Proof

For proof of main result of this paper we prpve the following steps.

(Step1) We can assume without loss of generality that $p \neq 2$. For if p=2, then G is easily seen to be S^4 or A^4; hence G is doubly transitive.

(Step2) Every element different from 1 of a Sylow p-subgroup of G is a product of two p-cycles. Every Sylow p-subgroup of G is semiregular and has order p.

Proof

Let P be a Sylow p-subgroup of G and $1 \neq x \in P.$ Since the order of P is a power of p, x consists of p-cycles and cycles of length 1. By 2.6 (theorem of Jordan) G has no p-cycles and x is therefore a product of two p-cycles. In particular, x moves every point, hence P is semiregular. By 2.11, $|P|$ is a divisor of 2p, thus $|P| = p$ because $p \neq 2$ by (Step1).

(Step3) If $g \in G$ and a is the order of g, then either a=p or (a, p) =1.

Proof

We assume that p divides a and $p \neq a$. Then $h = g^{\alpha/\rho} \neq 1$, thus h has order p. By (Step2), h is a product of two p-cycles. Therefore in g no cycle can appear whose length is prime to p, since h has only cycles of length p. If g were a 2p-cycle, G would contain the regular group <g>. which is cyclic and of composite order 2p. By 2.8 (theorem of Schur) G would be doubly transitive, which is not the case. g therefore has only p-cycles, hence has order p, which contradicts our assumption.

We again denote by $D_1, ..., D_r$ the different irreducible constituents of the permutation representation G^* of G. In addition let f_i be the degree of $D_i(i=1,..., r)$ and $let\ \chi_i = Tr(D_i)$ be the character of D_i. Let the numbering be chosen so that D_1 is the identity representation. We now prove:

Theorem

We can number the irreducible constituents D_i of G^* such that $f_2 = p$ and the representations $D_3, ..., D_r$ are conjugate. In particular $f_3 = ... = f_r = f,$ and f divides p-1.

Proof. 4(a)

By (Step2) there is an $x \in G$ which is the product of two p-cycles. Without loss of generality we may put x=(12…p) (p+1 … 2p). The characteristic polynomial of the permutation matrix x^* associated with x is $(z^p-1)^2$. Hence x^* has the eigenvalues 1, u,…,u^{p-1}, all with multiplicity 2, where u is a primitive pth root of unity. We wish to investigate how these eigenvalues are distributed among the $D_i(x)$. $D_t(x)$ has the eigenvalue 1 with multiplicity 1 and no others, since D_1 is the identity representation.

4(b) We now show that $f_i>1$ holds for $i \geq 2$. We assume $f_i=1$. Since G is not Abelian, but $D_i(G)$ is Abelian, D_i is not faithful. The normal subgroup N of all $n \in G$ with $D_i(n)=1$ is therefore different from 1 and henc (by 2.10) is transitive. Thus $D_i(N)=1$ and also $D_1(N)=1$, which by 2.11 cannot be the case. (Note: This argument is valid for all primitive non-Abelian groups.)

4(c) Let the numbering of the irreducible constituents $D_2,\ldots,D_r$ be chosen so that $D_2(x)$ has 1 as an eigenvalue. Because $f_2>1$ and since the eigenvalue 1 occurs for $*$ only twice, $D_2(x)$ has an eigenvalue different from 1 which without loss of generality may be assumed to be u.

4(d) All representations conjugate to a D_i are constituents of G* since G* is rational. D_2 is conjugate to itself, for otherwise a $D_i(x)$ with $i \geq 3$ would have the eigenvalue 1, which is impossible since $*$ has the eigenvalue 1 altogether only twice. Therefore $D_2(x)$ has the eigenvalues $1,u,\ldots,u^{p-1}$, all with multiplicity 1, for if an eigenvalue,say u, appeared in $D_2(x)$ with multiplicity 2, then because of the rationality of χ_2 so would $u^2,\ldots,u^{p-1}$. Then, however, D_1 and D_2 would be the only irreducible constituents of G*, and by 2.14 G would be doubly transitive. Hence we obtain $f_2=p$.

4(e) The remaining eigenvalues $u,\ldots u^{p-1}$ (all with multiplicity 1) of $*$ are divided among the remaining representations $D_3,\ldots,D_r$. We now prove that these representations are conjugate to each other and therefore have the same degree f . For r=3 there is nothing more to show. We assume $r \geq 4$. It suffices (without loss of generality) to prove that D_3 is conjugate to D_4. Let u be an eigenvalue of $D_3(x)$, u^s one of $D_4(x)$ $(1 < s \leq p-1)$. Because (p, |G|/p)=1 (by (Step2)) there is an m which is a solution of the two congruences $m \equiv s(p)$ and $m \equiv 1(|G|/p)$. This yields (m, |G|)=1, and therefore an irreducible constituent D_i of G* conjugate to D_3 is defined by $\chi_i(g) = \chi_3(g^m)(g \in G)$. Because of the rationality of D_1 and D_2, we have $i \geq 3$. In addition, $u^m=u^s$ is an eigenvalue of $D_i(x)$. It is also an eigenvalue of $D_4(x)$. Since u^s occurs altogether only once in $D_3,\ldots,D_r$ we have $D_i=D_4$, hence D_4 conjugate to D_3.

4(f) In particular we obtion $p-1 \equiv 0$(f). In addition, 4(d) and 4(e) show that all the D_i occur only with multiplicity 1: $e_1=\ldots=e_r=1$.

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